IMPERIAL

Causal Discovery for Trustworthy AI

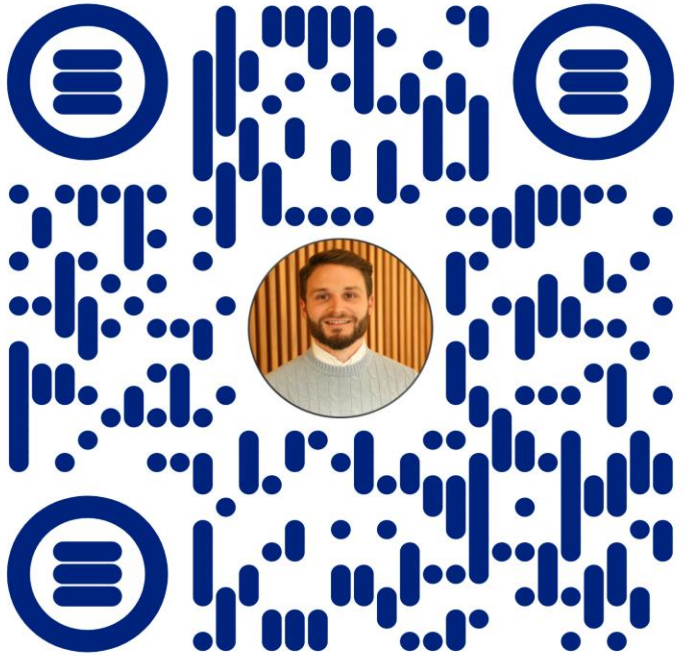
Fabrizio Russo

Ethics, Fairness and Explanation in AI
19/02/2026 Guest Lecture

My Journey

Across Finance And Artificial Intelligence

2009-2012	Rome	– Tor Vergata: BSc Economics & Finance
	Madrid	– Autónoma
2013-2014	London	– Learning & Exploring
2014-2015	London	– General Electric Capital: Credit Risk Analyst
2015-2020	London	– 4most Europe: Consultant – Head of Data Science
2016-2018	London	– Birkbeck College: MSc Applied Statistics
2020-2024	London	– Imperial College: PhD Safe and Trusted AI
2025-2026	London	– Imperial College: Research Associate
2026 June	London	– Imperial College: Research Fellow

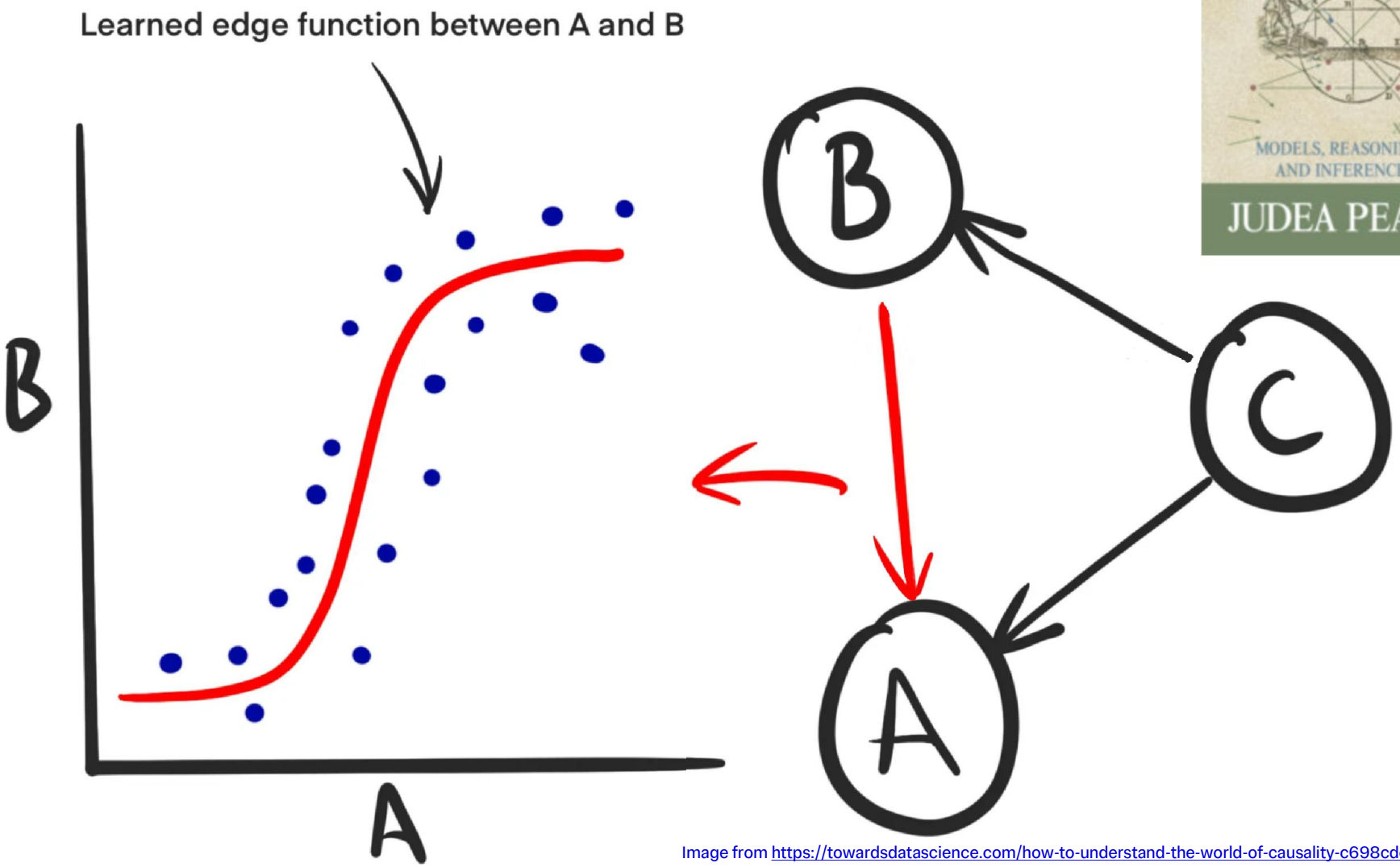


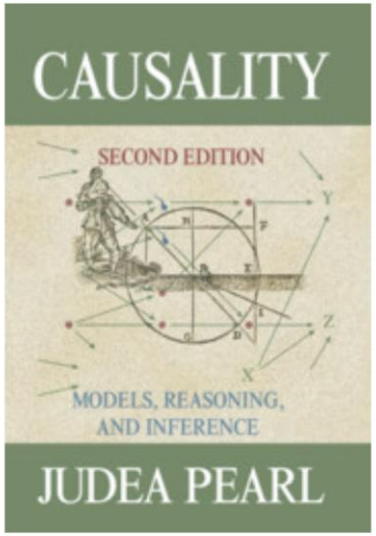
Agenda

1. **Motivation & Background**
   i. Causal Models
   ii. Causal Discovery
2. **Causal Graphs for Contestable Neural Networks**
3. **Argumentative Causal Discovery**
   i. LLM as Imperfect Experts
4. **Recap & Conclusion**

Structural Causal Model (Pearl, 2009)

Causal Graphs + Structural Equation





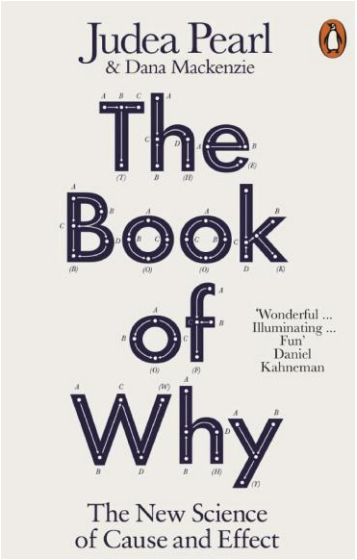


Image from https://towardsdatascience.com/how-to-understand-the-world-of-causality-c698cdc9f27c

Prediction vs Intervention

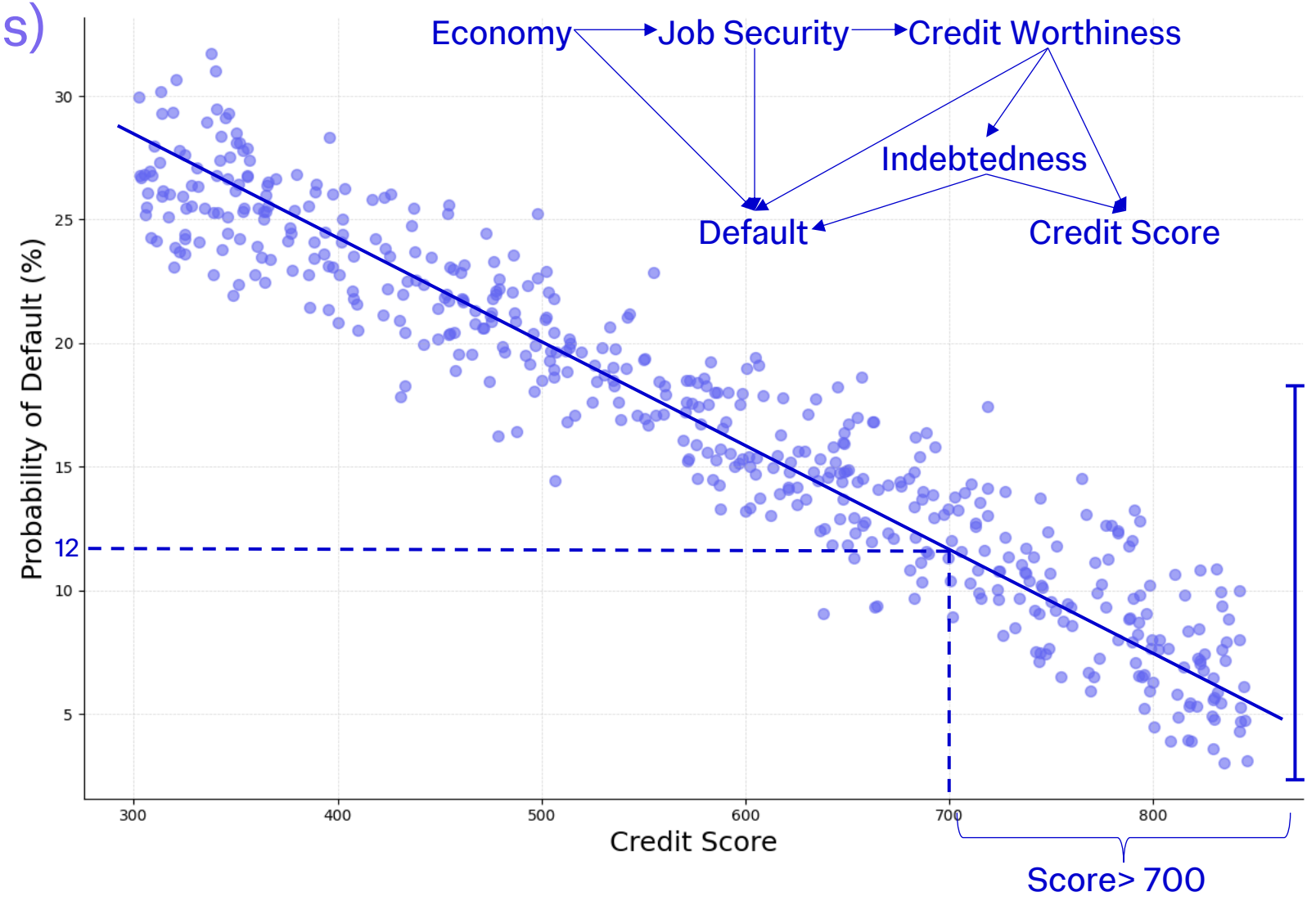
Causal Assumptions (graphs)

Predictive Models

- What if we *observe* a credit score of 700?
- What if we *observe* a PD of 12%?

Causal Models

- What if we accept only customers with 700+ score?
- What if we *do* (perform an action/intervention in an environment)?



Prediction vs Intervention

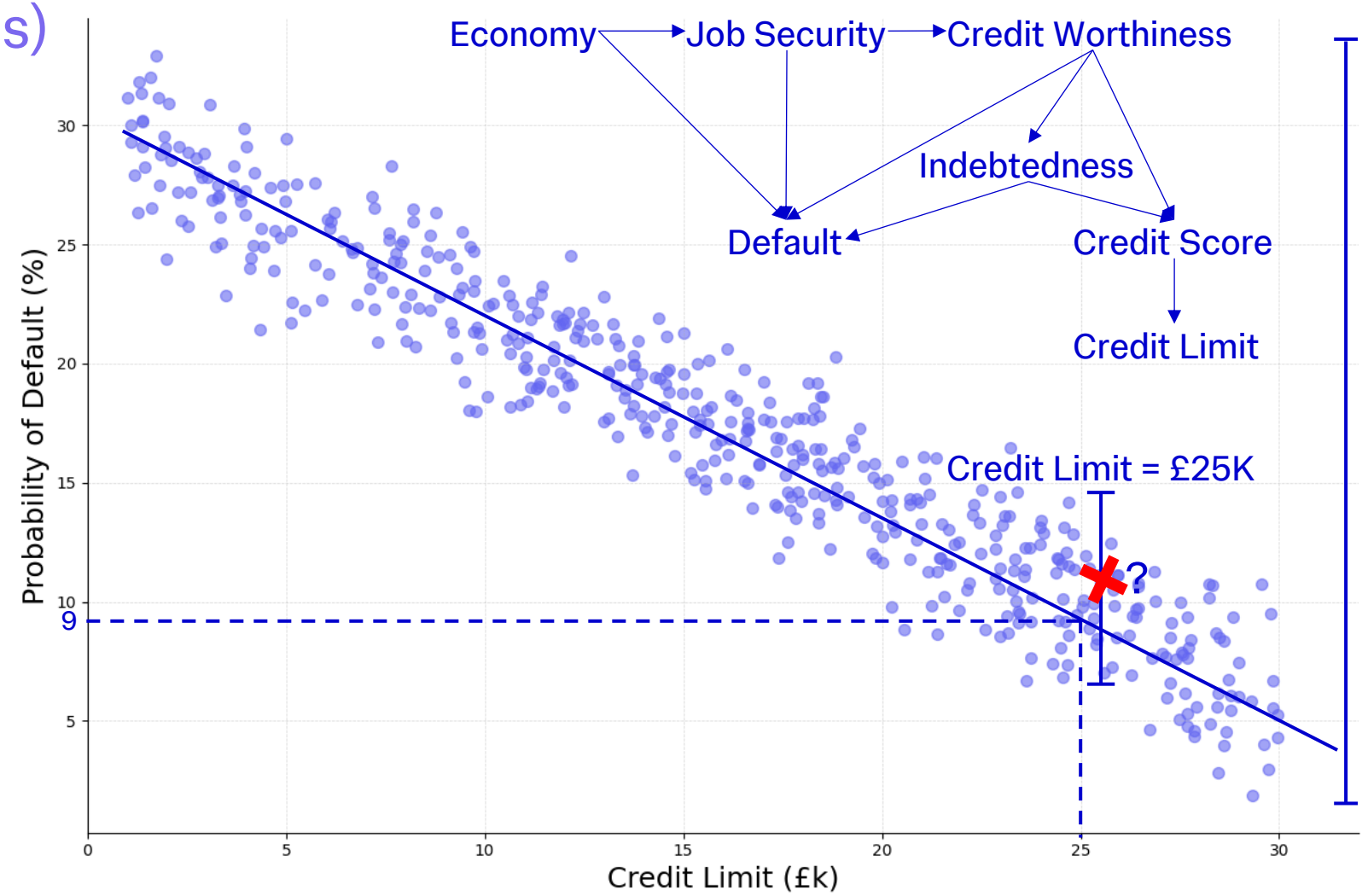
Causal Assumptions (graphs)

Predictive Models

- What if we observe a credit limit of £25k?
- What if we observe a PD of 9%?

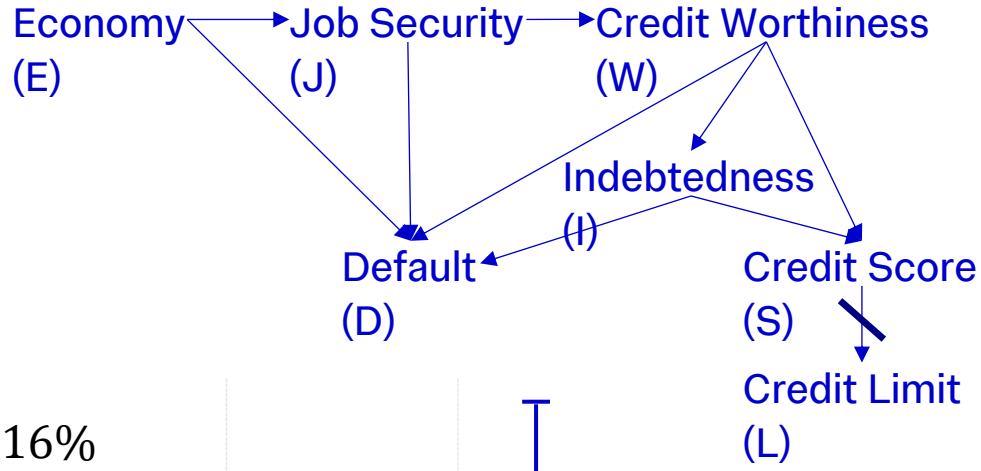
Causal Models

- What if we *intervene* on Credit Limit?
- What's the *causal* effect on the PD?



Causal Models

Structural Equations



Exogenous

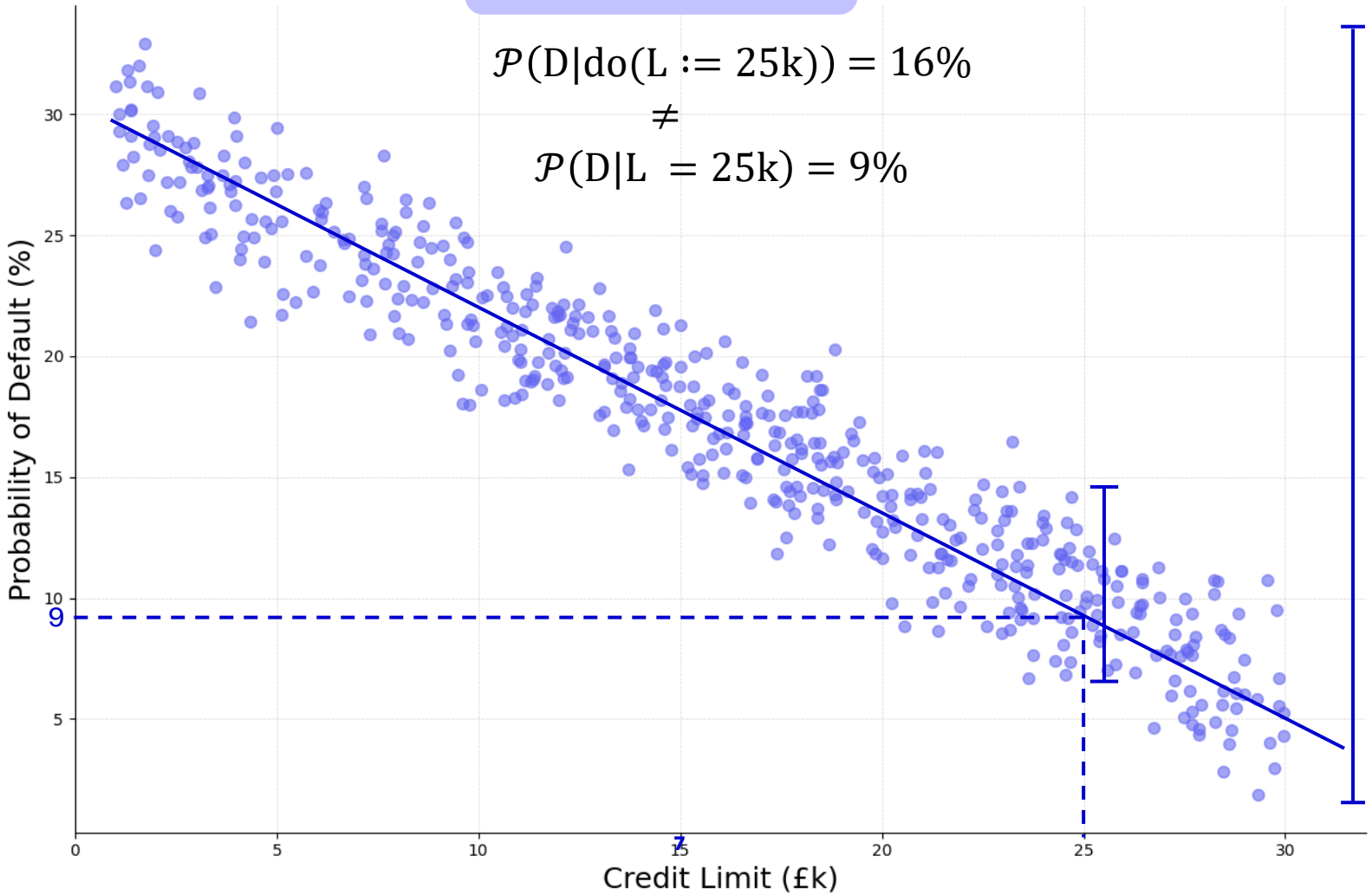
$$E \coloneqq f_{\mathrm{E}}(U_{\mathrm{E}})$$

Endogenous:

$$\mathrm{J} \coloneqq f_{\mathrm{J}}(\mathrm{E}, U_{\mathrm{J}})$$
$$W \coloneqq f_{W}(\mathrm{J}, U_{W})$$
$$\mathrm{I} \coloneqq f_{\mathrm{I}}(\mathrm{W}, U_{\mathrm{I}})$$
$$D \coloneqq f_{\mathrm{D}}(\mathrm{E}, \mathrm{J}, \mathrm{W}, U_{\mathrm{D}})$$
$$\mathrm{S} \coloneqq f_{\mathrm{S}}(\mathrm{W}, \mathrm{I}, U_{S})$$
$$\mathrm{L} \coloneqq 25\mathrm{k}$$

$$\mathrm{do}(\mathrm{L} \coloneqq 25\mathrm{k})$$

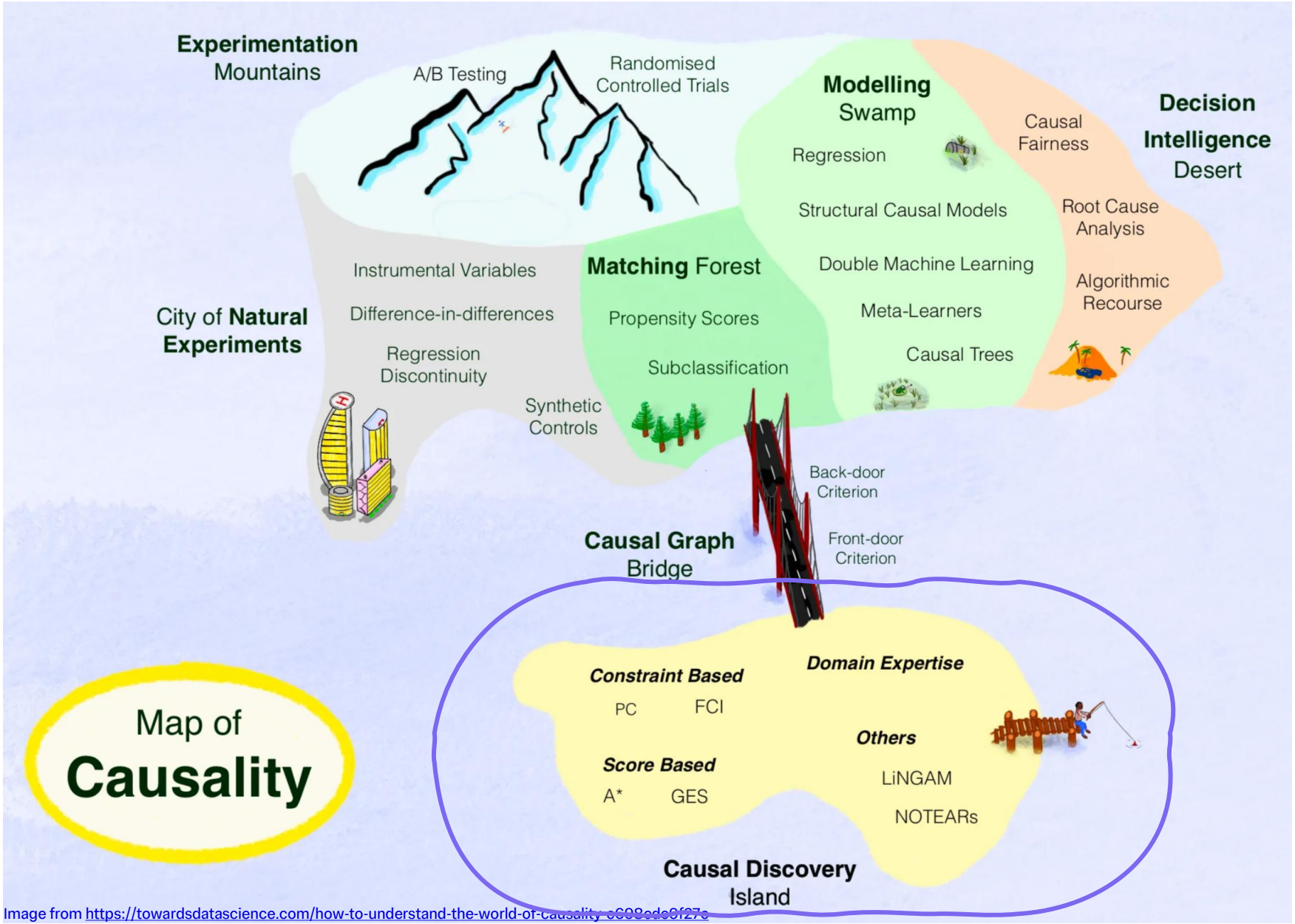
$$\mathcal{P}(\mathrm{D}|\mathrm{do}(\mathrm{L} \coloneqq 25\mathrm{k})) = 16\%$$
$$\neq$$
$$\mathcal{P}(\mathrm{D}|\mathrm{L} = 25\mathrm{k}) = 9\%$$



Machine Learning vs Causal Models

High-Level Comparison

Attribute	$\mathcal{P}(\mathrm{D}\|\mathrm{L} = 25\mathrm{k})$ Machine Learning	$\mathcal{P}(\mathrm{D}\|\mathrm{do}(\mathrm{L} := 25\mathrm{k}))$ Causal Models
Interpretability	Limited	High
Predictive Accuracy	High	Moderate
Generalisation	Moderate	High
Actionable Insights	Limited	High
Data Requirements	High	Moderate
Scalability	High	Low
Expert Knowledge	Moderate	High



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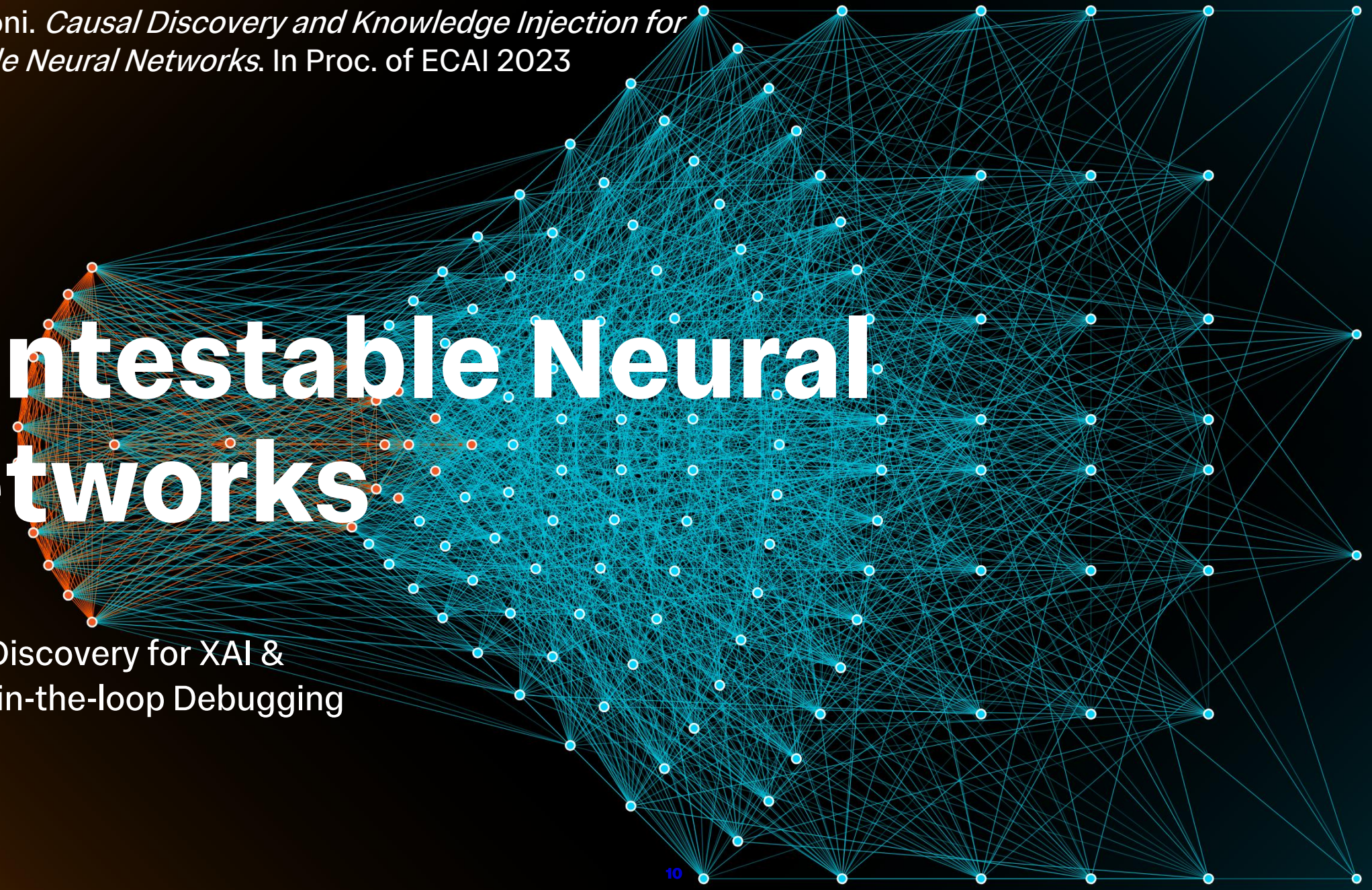
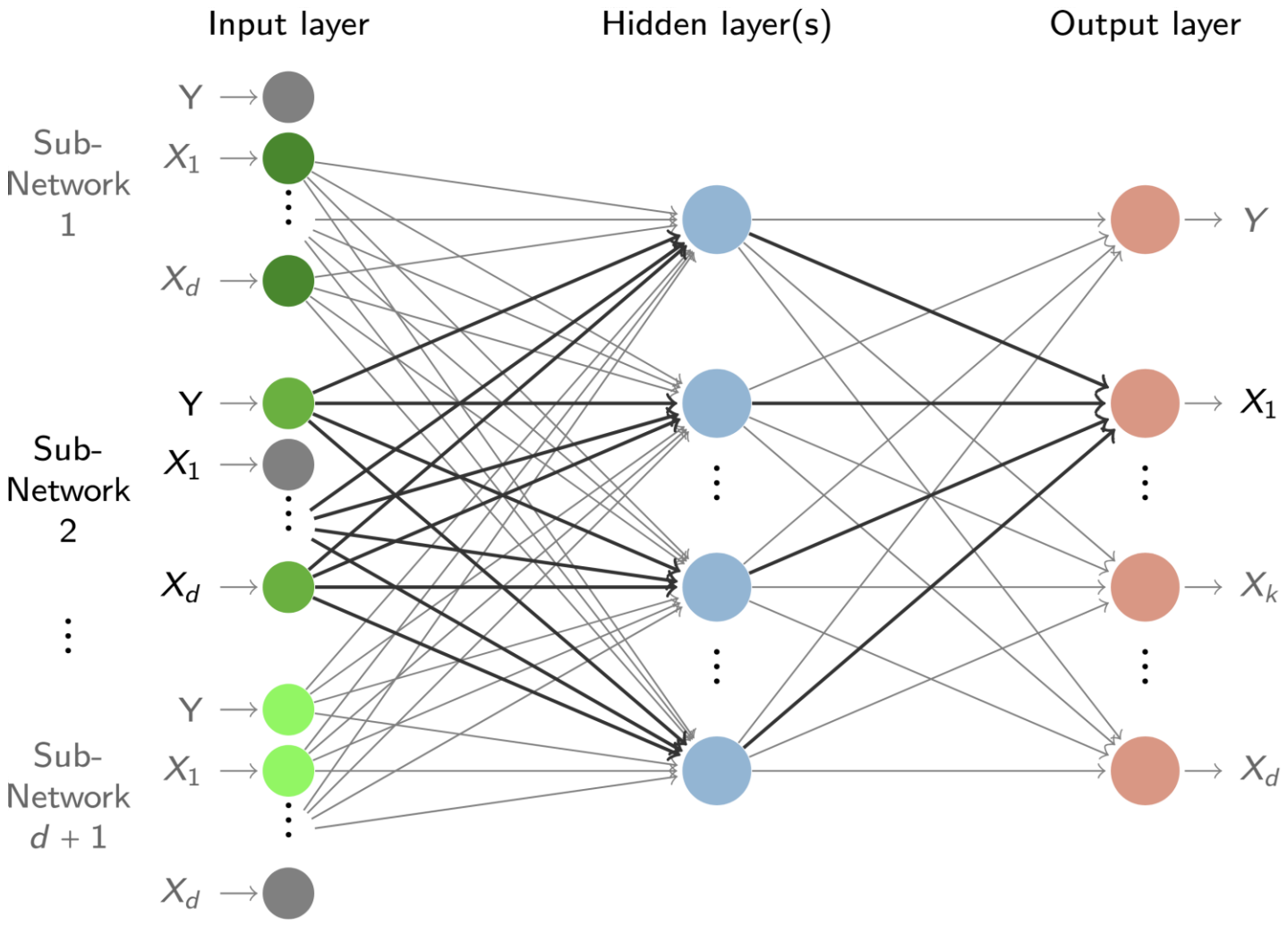
Russo & Toni. *Causal Discovery and Knowledge Injection for Contestable Neural Networks*. In Proc. of ECAI 2023

Contestable Neural Networks



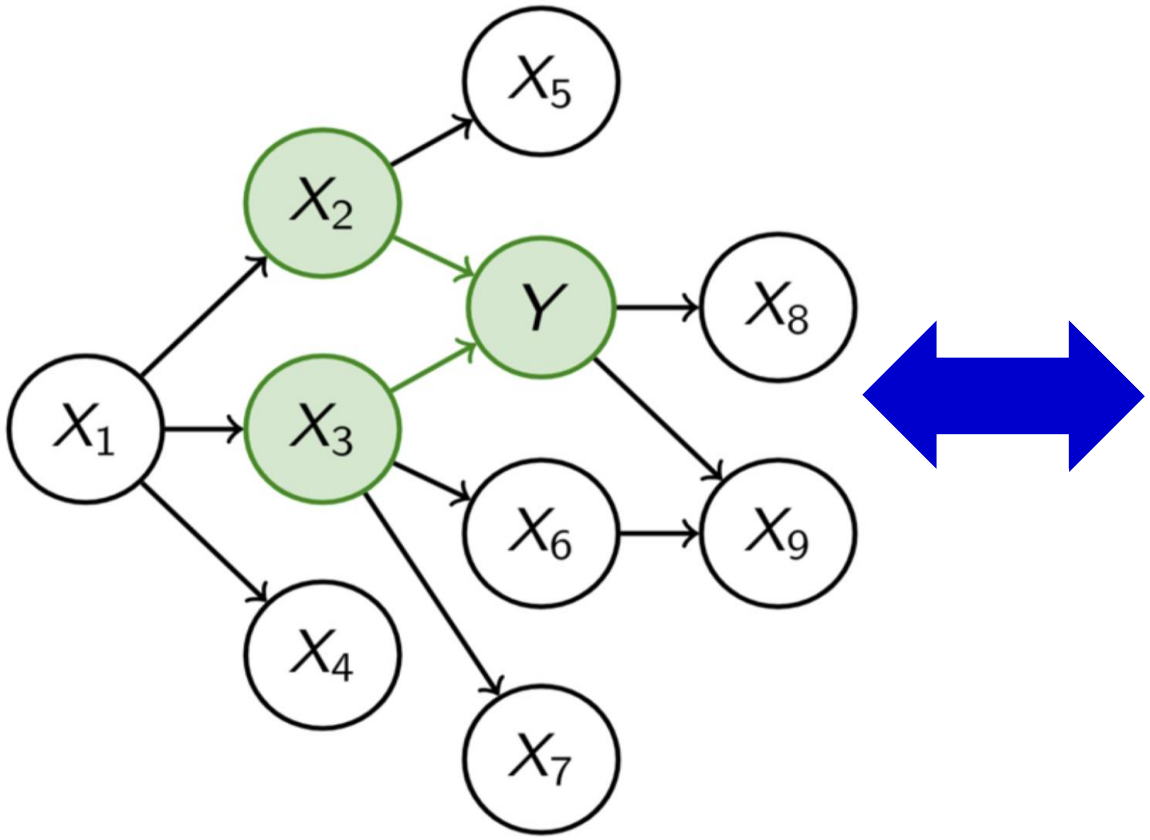
Causal Discovery for XAI &
Human-in-the-loop Debugging

Joint Neural Network Structure
(Kyono, Zhang and van der Schaar 2020)



Objective

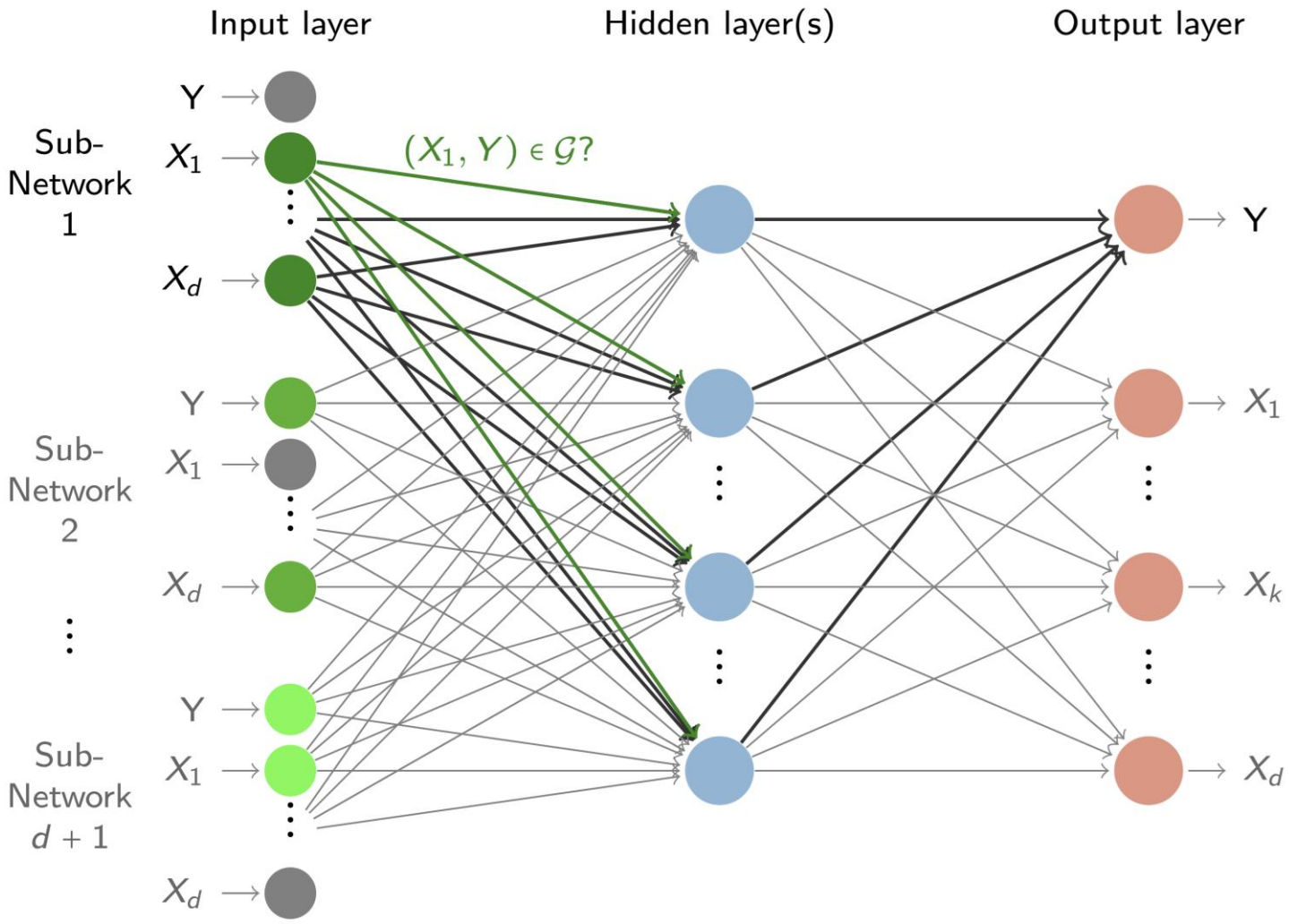
Capture Causal Relations



	Y	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
Y	0.0	0.005	0.017	0.008	0.002	0.042	0.02	0.005	0.059	0.05
X_1	0.006	0.0	0.063	0.054	0.068	0.009	0.006	0.013	0.006	0.008
X_2	0.088	0.036	0.0	0.022	0.019	0.124	0.008	0.011	0.006	0.008
X_3	0.087	0.034	0.021	0.0	0.024	0.005	0.107	0.104	0.006	0.009
X_4	0.009	0.032	0.02	0.023	0.0	0.01	0.013	0.01	0.005	0.005
X_5	0.026	0.006	0.017	0.004	0.004	0.0	0.012	0.002	0.005	0.018
X_6	0.025	0.006	0.008	0.011	0.005	0.017	0.0	0.014	0.002	0.114
X_7	0.029	0.003	0.007	0.011	0.002	0.024	0.029	0.0	0.011	0.01
X_8	0.036	0.002	0.004	0.003	0.004	0.006	0.009	0.006	0.0	0.006
X_9	0.024	0.003	0.003	0.004	0.003	0.005	0.079	0.01	0.004	0.0

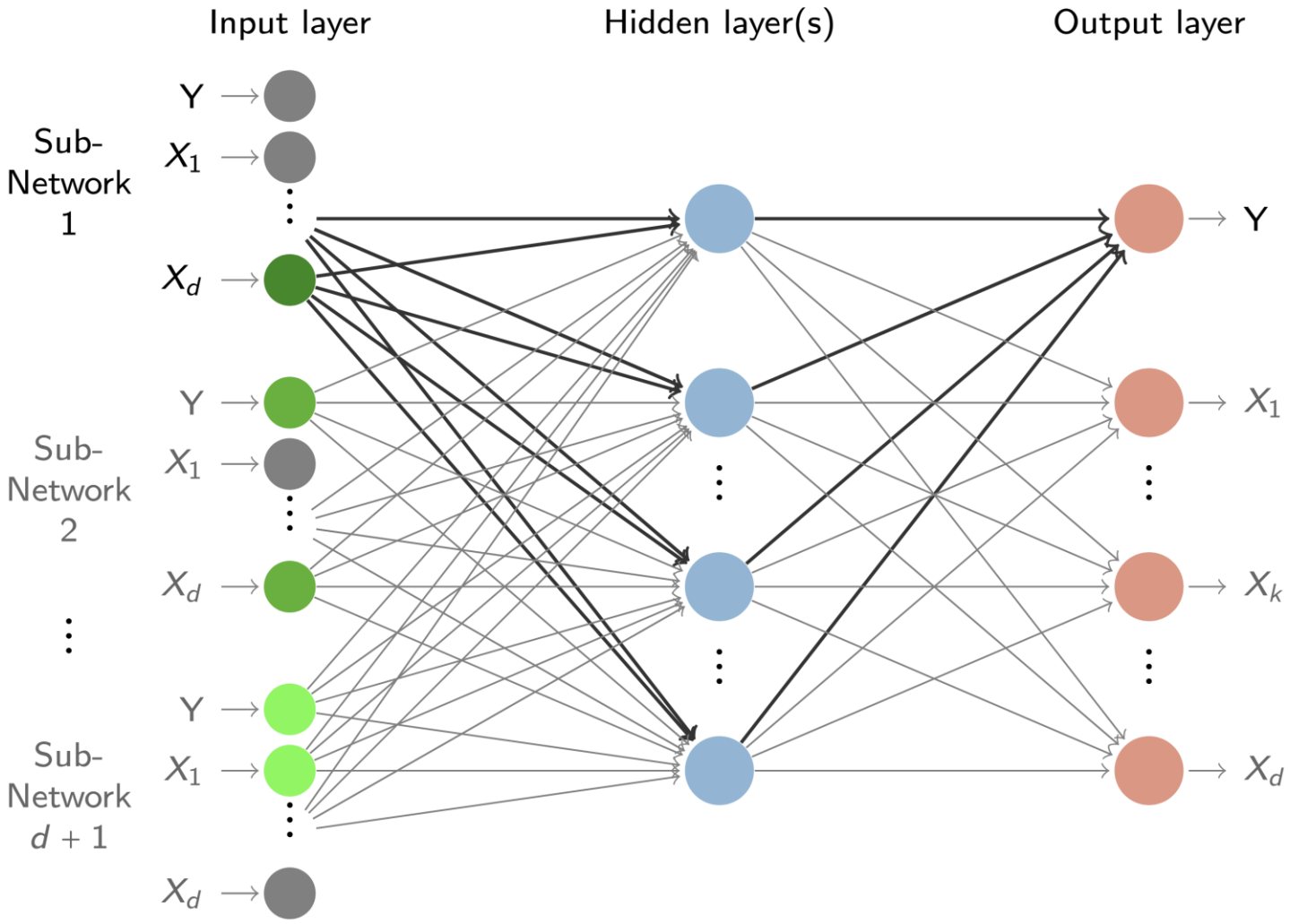
Encode Causality in the Network Structure

(Kyono, Zhang and van der Schaar 2020)



Encode Causality in the Network Structure

(Kyono, Zhang and van der Schaar 2020)



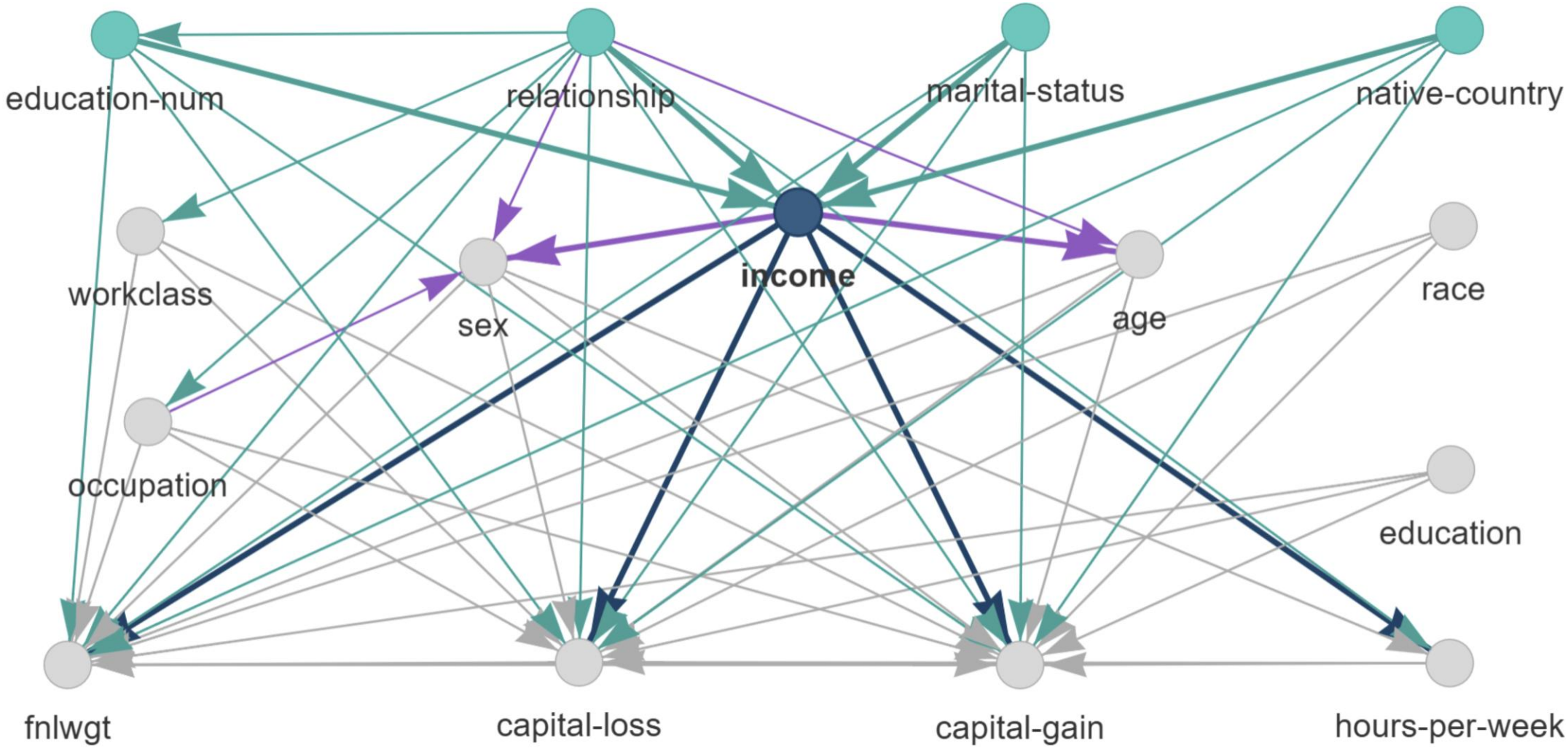
Income Prediction Case Study

Adult Dataset (Becker and Kohavi, 1996)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	
(1)			0.0	0.2				0.1					36.1	3.7	21.3	income
(2)													1.6	0.3	11.5	race
(3)								0.1					1.9	0.4	16.3	sex
(4)													0.8	0.1	2.0	age
(5)	0.0												1.1	0.2	4.9	native-country
(6)			0.0										0.6	0.2	4.0	occupation
(7)													1.2	0.3	6.4	workclass
(8)													0.8	0.1	3.6	hours-per-week
(9)													0.5	0.2	5.5	education
(10)	0.0												2.7	0.4	6.9	education-num
(11)	0.1												2.0	0.3	18.1	marital-status
(12)	0.0		0.0	0.1		0.0	0.0	0.1		0.0			2.6	0.5	15.0	relationship
(13)															0.1	capital-gain
(14)													0.2		0.5	capital-loss
(15)																fnlwgt

Income Prediction Case Study

Adult Dataset (Becker and Kohavi, 1996)



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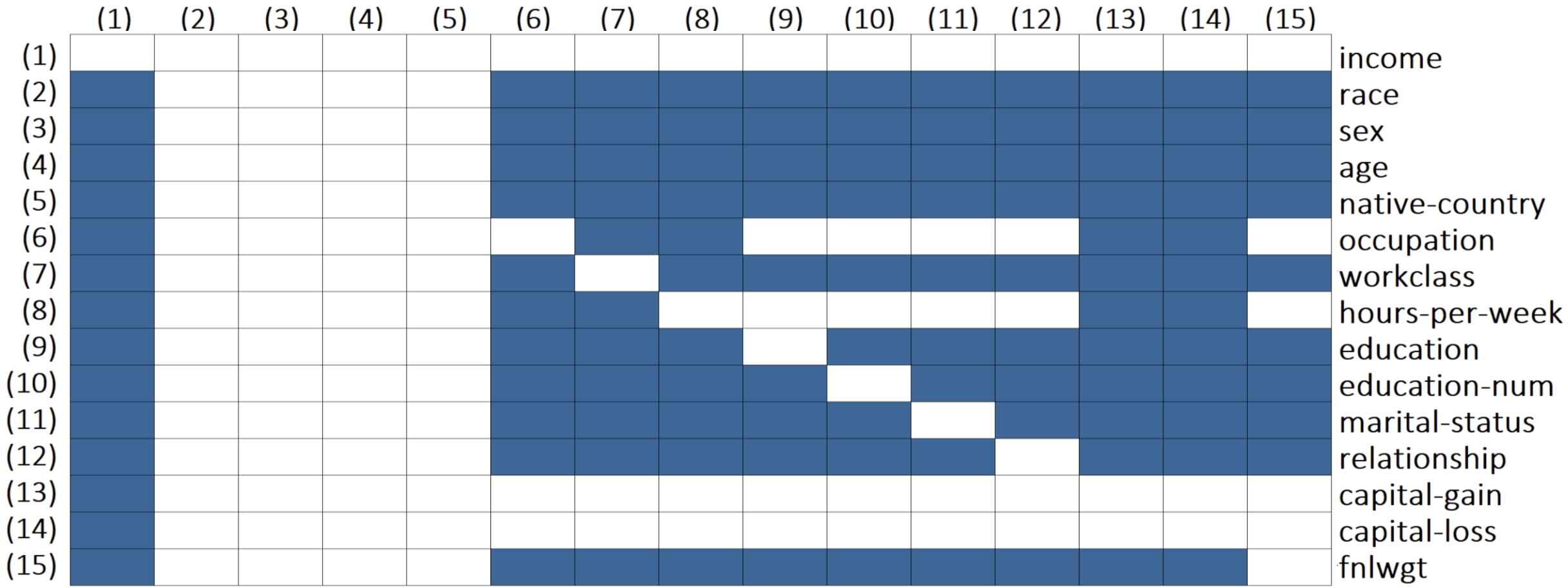

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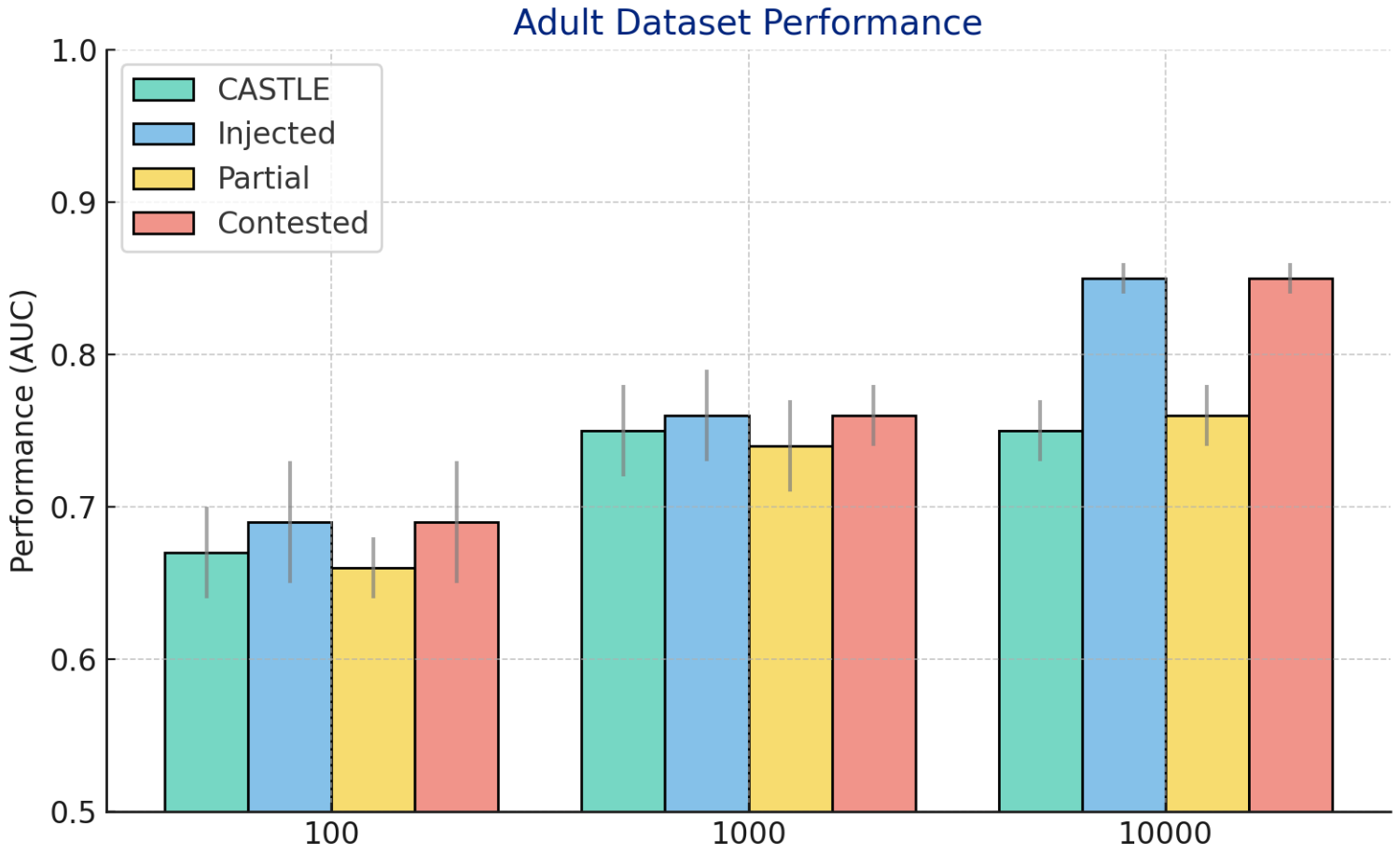


Takeaways

Causal Graphs to...

Explain

Contest

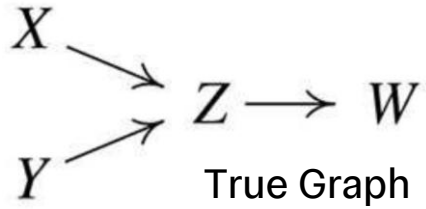
Improve

Russo & Toni. *Causal Discovery and Knowledge Injection for Contestable Neural Networks*. In Proc. of ECAI 2023

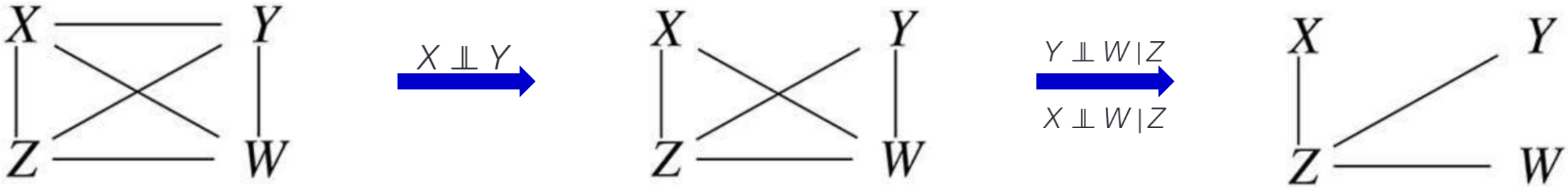
How do we reliably build causal graphs from data?

The Peter-Clark Algorithm

(Spirtes et al, 1993) Example from Glymour et al, 2019

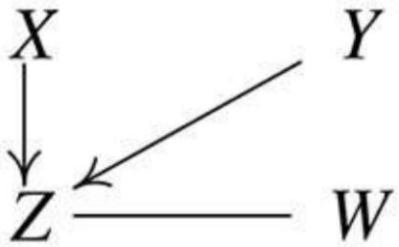


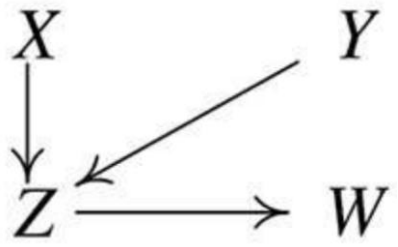
True Graph



$X \perp\!\!\!\perp Y$

$X \not\perp\!\!\!\perp Y \mid Z$





Income Prediction Example

$E \not\perp\!\!\!\perp I$	$p = 0.00$	$\mathcal{S} = 1.00$
$E \not\perp\!\!\!\perp O$	$p = 0.00$	$\mathcal{S} = 1.00$
$R \not\perp\!\!\!\perp O$	$p = 0.00$	$\mathcal{S} = 1.00$
$O \not\perp\!\!\!\perp I$	$p = 0.00$	$\mathcal{S} = 1.00$
$R \perp\!\!\!\perp E$	$p = 0.46$	$\mathcal{S} = 0.71$
$R \not\perp\!\!\!\perp I$	$p = 0.05$	$\mathcal{S} = 0.52$
$E \not\perp\!\!\!\perp I \mid \{R\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$E \not\perp\!\!\!\perp I \mid \{O\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$E \not\perp\!\!\!\perp O \mid \{R\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$R \not\perp\!\!\!\perp O \mid \{I\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$E \not\perp\!\!\!\perp O \mid \{I\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$R \not\perp\!\!\!\perp O \mid \{E\}$	$p = 0.00$	$\mathcal{S} = 0.50$

$O \not\perp\!\!\!\perp I \mid \{E\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$O \not\perp\!\!\!\perp I \mid \{R\}$	$p = 0.00$	$\mathcal{S} = 0.50$
$R \perp\!\!\!\perp E \mid \{O\}$	$p = 0.53$	$\mathcal{S} = 0.38$
$R \not\perp\!\!\!\perp I \mid \{O\}$	$p = 0.03$	$\mathcal{S} = 0.35$
$R \perp\!\!\!\perp E \mid \{O\}$	$p = 0.33$	$\mathcal{S} = 0.32$
$R \perp\!\!\!\perp E \mid \{I\}$	$p = 0.05$	$\mathcal{S} = 0.25$
$R \perp\!\!\!\perp E \mid \{O, I\}$	$p = 0.39$	$\mathcal{S} = 0.00$
$R \not\perp\!\!\!\perp I \mid \{E, O\}$	$p = 0.00$	$\mathcal{S} = 0.00$
$E \not\perp\!\!\!\perp O \mid \{R, I\}$	$p = 0.00$	$\mathcal{S} = 0.00$
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Income Prediction Example

True Causal Graph

Majority-PC (Colombo and Maathias, 2012)

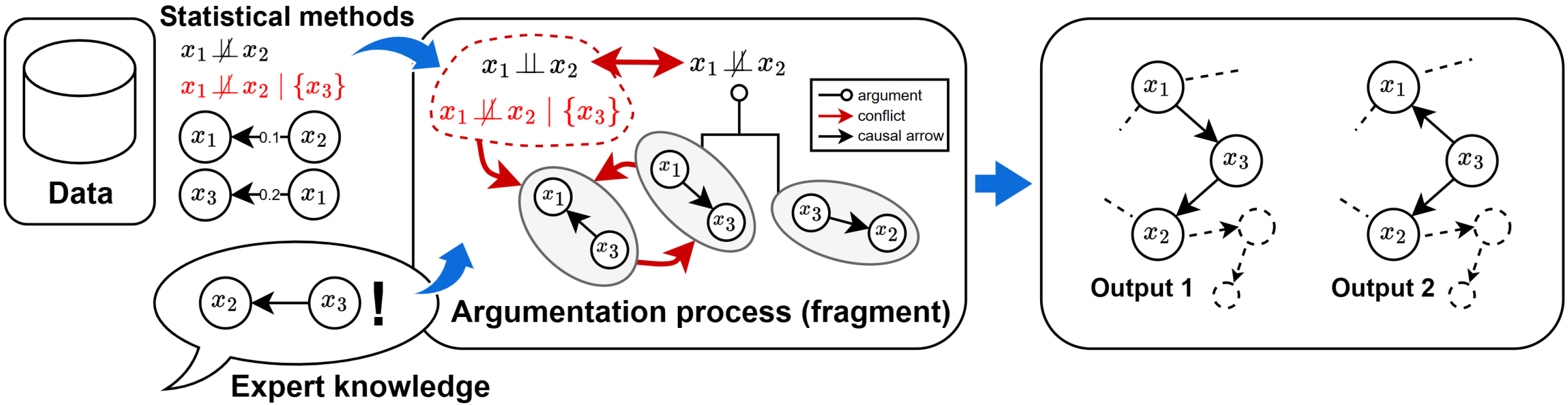
No Directed Acyclic Graph (DAG) is compatible with *all* the tests at the same time

Argumentative Causal Discovery

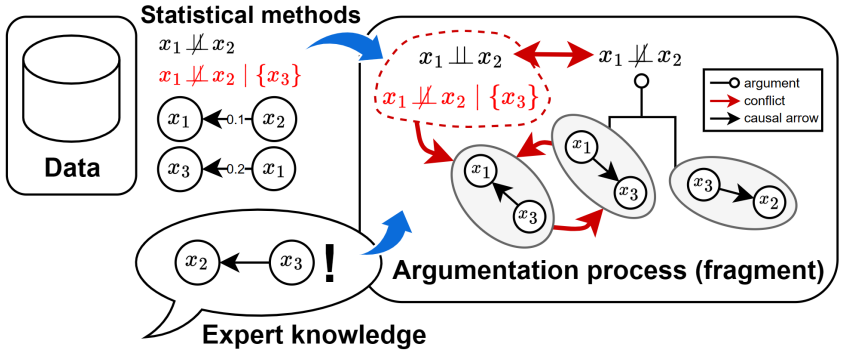
A Debate about Causality

Fabrizio Russo (Imperial College London)
Anna Rapberger (Imperial College London)
Francesca Toni (Imperial College London)

In Proceedings of the 21st International Conference on Principles of Knowledge Representation and Reasoning — Reasoning, Learning & Decision-Making Track. Pages 938–949.



Causal ABA (Assumption-based Argumentation)



Assumptions

$$\{arr_{xy} \mid x, y \in \mathbf{V}, x \neq y\} \cup \{noe_{xy} \mid x, y \in \mathbf{V}, x \neq y\}$$

$$\{(x \perp\!\!\!\perp y \mid \mathbf{Z}) \mid \mathbf{Z} \subseteq \mathbf{V}, x, y \in \mathbf{V} \setminus \mathbf{Z}, x \neq y\}.$$

$$\{\mathbf{bp_{p|Z}} \mid \mathbf{p} \text{ is a x-y-path}, \mathbf{Z} \subseteq \mathbf{V} \setminus \{x, y\}\}$$

Rules

- $\overline{a} \leftarrow b,\ a \neq b,\ a, b \in \{arr_{xy}, arr_{yx}, noe_{xy}\},\ x, y \in \mathbf{V} \implies$ **Can only choose one assumption**
- $\overline{arr_{x_i x_{i+1}}} \leftarrow arr_{x_1 x_2}, \ldots, arr_{x_{k-1} x_k}$ for each sequence $x_1 \ldots x_k$ with $x_1 = x_k$, for each $1 \leq i < k \implies$ **Cycles are not allowed**

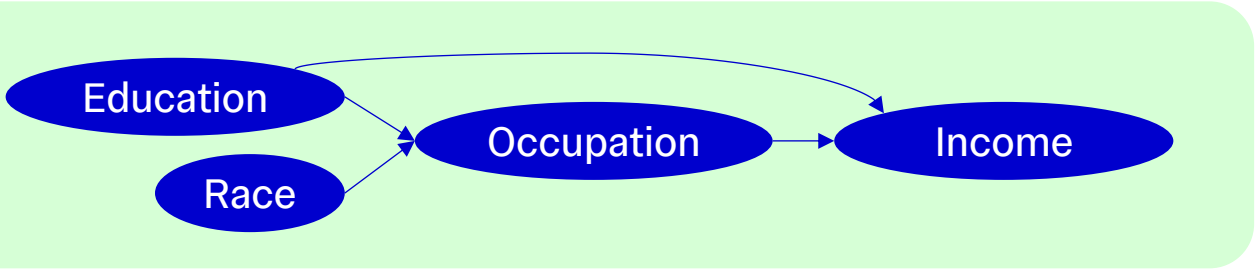
$dpath_{xy} \leftarrow arr_{xy}$ $\quad$ $dpath_{xz} \leftarrow dpath_{xy}, arr_{yz}$

$e_{xy} \leftarrow arr_{xy}$ $\quad$ $e_{xy} \leftarrow arr_{yx}$ $\quad$ $\overline{noe_{xy}} \leftarrow e_{xy}$

$\overline{x \perp\!\!\!\perp y \mid \mathbf{Z}} \leftarrow p_{\mathbf{Z}}$

- $x \perp\!\!\!\perp y \mid \mathbf{Z} \leftarrow bp_{p_1|\mathbf{Z}}, \ldots, bp_{p_k|\mathbf{Z}}$ where $p_1, \ldots, p_k$ denote all paths between x and y;
- $ap_{p|\mathbf{Z}} \leftarrow p$ for each $\mathbf{Z}$-active p

Income Prediction Example



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~~$E \not\perp\!\!\!\perp O \mid \{R, I\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$R \not\perp\!\!\!\perp I \mid \{E, O\}$~~	~~$p = 0.00$~~	~~$\mathcal{S} = 0.00$~~
~~$R \not\perp\!\!\!\perp O \mid \{E, I\}$~~	~~$p = 0.03$~~	~~$\mathcal{S} = 0.00$~~

Argumentative Causal Discovery

Robust & Interactive

Sound & Complete

Robust to Errors but
Computationally Demanding

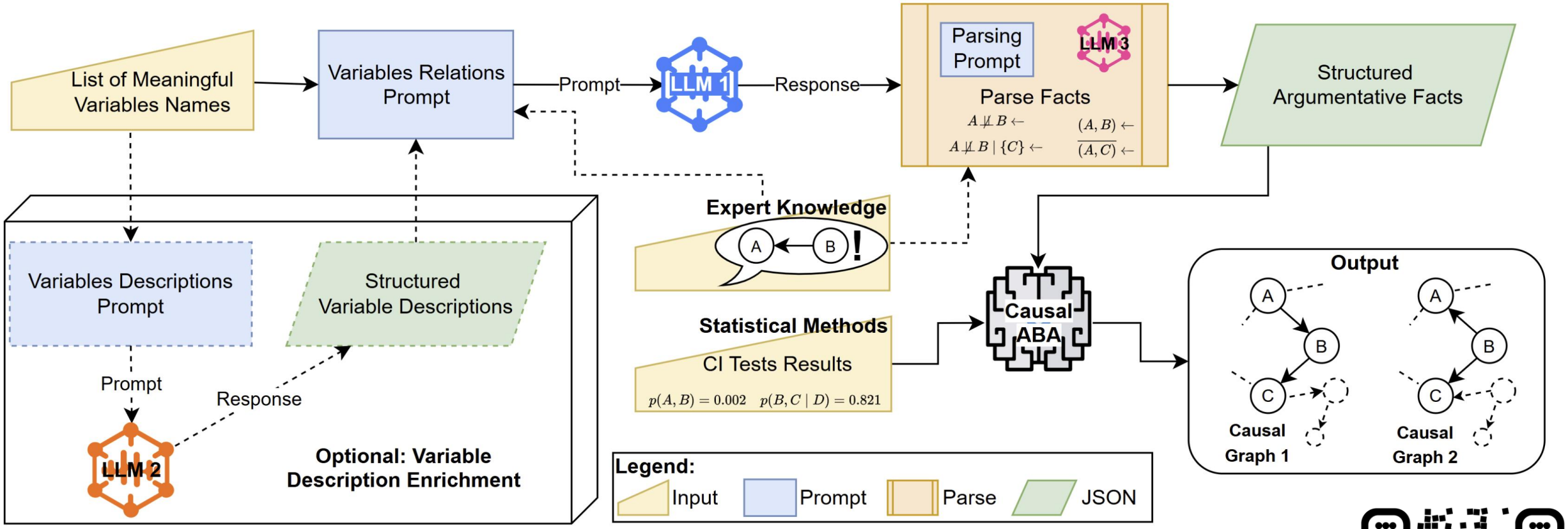
Data Check &
Stakeholder Engagement

Russo, Rapberger & Toni. *Argumentative Causal Discovery*.
In Proc. of KR 2024.

Causal ABA Ensemble

LLMs as Imperfect Experts

Li, Z. and Russo, F. *Leveraging Large Language Models for Causal Discovery: a Constraint-based, Argumentation-driven Approach*. Arxiv 2602.16481



Causal Discovery Literature

From the 90s

frontiers in Genetics

REVIEW
published: 04 June 2019
doi: 10.3389/fgene.2019.00524

Check for updates

Review of Causal Discovery Methods Based on Graphical Models

Clark Glymour, Kun Zhang and Peter Spirtes*

Department of Philosophy, Carnegie Mellon University, Pittsburgh, PA, United States

A fundamental task in various disciplines of science, including biology, is to find underlying causal relations and make use of them. Causal relations can be seen if interventions are properly applied; however, in many cases they are difficult or even impossible to conduct. It is then necessary to discover causal relations by analyzing statistical properties of purely observational data, which is known as causal discovery or causal structure search. This paper aims to give a introduction to and a brief review of the computational methods for causal discovery that were developed in the past three decades, including constraint-based and score-based methods and those based on functional causal models, supplemented by some illustrations and applications.

Keywords: directed graphical causal models, causal discovery, conditional independence, statistical independence, structural equation models, non-Gaussian distribution, non-linear models

D'ya Like DAGs? A Survey on Structure Learning and Causal Discovery

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MATTHEW J. VOWELS, NECATI CIHAN CAMGOZ, and RICHARD BOWDEN,
CVSSP, University of Surrey, U.K.

Causal reasoning is a crucial part of science and human intelligence. In order to discover causal relationships from data, we need structure discovery methods. We provide a review of background theory and a survey of methods for structure discovery. We primarily focus on modern, continuous optimization methods, and provide reference to further resources such as benchmark datasets and software packages. Finally, we discuss the assumptive leap required to take us from structure to causality.

CCS Concepts: • **Mathematics of computing** → **Causal networks**; • **Computing methodologies** → **Machine learning**; **Causal reasoning and diagnostics;**

Additional Key Words and Phrases: Causality, causal discovery, directed acyclic graphs, DAGs, structure learning, survey

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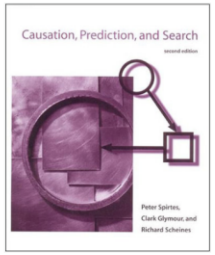

Adaptive Computation And Machine Learning Series

Causation, Prediction, and Search (Second Edition)

By Peter Spirtes, Clark Glymour, Richard Scheines

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Recap
& Conclusion

Causal Models

- Causal Discovery
- Structural Equations

Contestable NN

- Method & Process to align NNs to SMEs' causal view

Causal ABA

- Method to increase consistency guarantees and allow integrations and contestability

IMPERIAL

Questions?

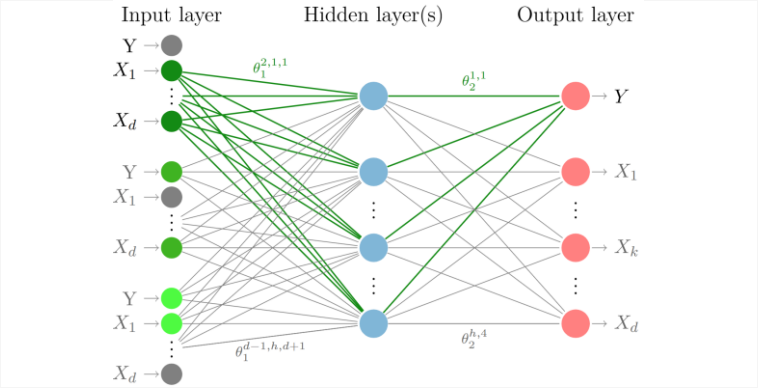
fabrizio@imperial.ac.uk

Appendix

Causal Discovery for Trustworthy AI

Methods

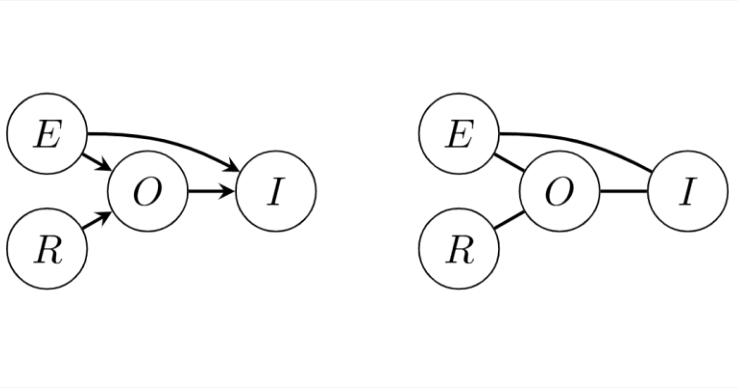
Contestable Neural Networks



Russo & Toni. *Causal Discovery and Knowledge Injection for Contestable Neural Networks*. In Proc. of ECAI 2023

- Learn a causal Graph while fitting a neural network
- Expose the learned graph to SMEs and allow them to modify it
- Inject the graph back into the NN

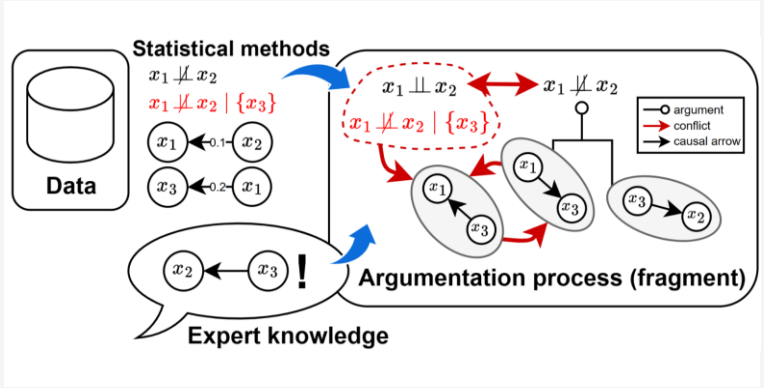
Causal Discovery with Shapley Values



Russo & Toni. *Shapley-PC: Constraint-based Causal Structure Learning with a Shapley Inspired Framework*. In Proc. of CLeaR 2025

- Learn a causal graph with asymptotic guarantees
- Identify colliders given a skeleton using Shapley Values
- Embed the decision rule into the PC algorithm

Argumentative Causal Discovery



Russo, Rapberger & Toni. *Argumentative Causal Discovery*. In Proc. of KR 2024.

- Represent DAGs and d-Separation as a debate
- Allow external knowledge to be input into the framework
- Guarantee consistency of output DAG with a subset of the input

Income Prediction Case Study

Adult dataset (Becker and Kohavi, 1996) UCI repository

Adult Dataset ($\|V\| = 14$)				
Data (N)	**CASTLE** $\|E\| = 210$	**Injected** $\|E\| = 46$	**Partial** $\|E\| = 116$	**Contested** $\|E\| = 30$
100	0.67 (0.03)	**0.69** .	0.66	**0.69** .
500	0.72 (0.04)	**0.74** *	0.71	**0.74** *
1000	0.75 (0.03)	**0.76**	0.74	**0.76**
2000	0.74 (0.03)	**0.77** ***	**0.76** *	**0.77** ***
5000	0.75 (0.03)	**0.79** ***	**0.76**	**0.79** ***
10000	0.75 (0.02)	**0.85** ***	**0.76** .	**0.85** ***
20000	0.76 (0.02)	**0.86** ***	**0.77** .	**0.86** ***

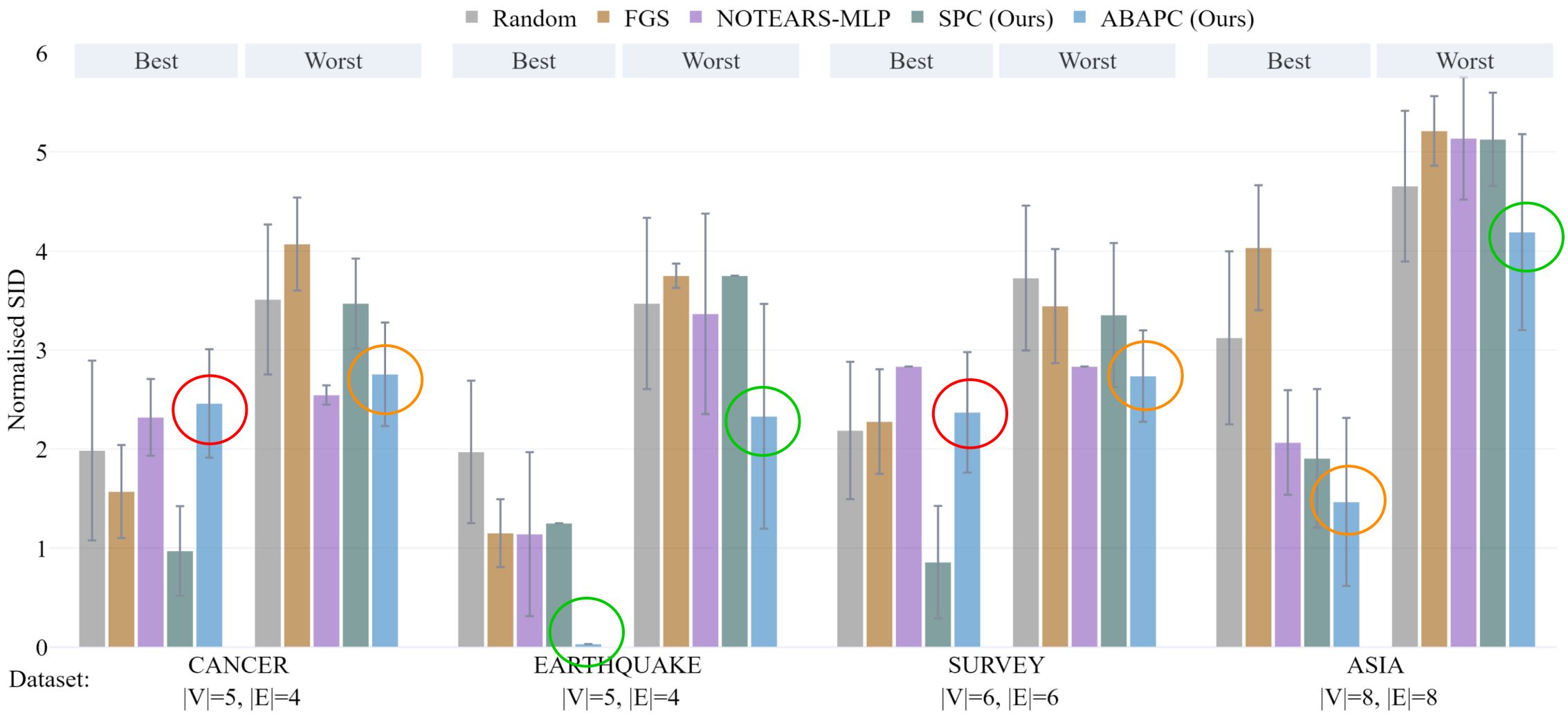
More Datasets

From Credit Risk to House Prices

Metric	Data & Method	N=100	N=500	N=1K	N=2K	N=5K	N=10K	N=20K
AUC ↑	**Adult** ($\|V\| = 14$)							
	CASTLE ($\|E\| = 210$)	0.67±0.03	0.72±0.04	0.75±0.03	0.74±0.03	0.75±0.03	0.75±0.02	0.76±0.02
	Injected ($\|E\| = 46$)	**0.69.**	**0.74***	**0.76**	**0.77*****	**0.79*****	**0.85*****	**0.86*****
	Partial ($\|E\| = 116$)	0.66	0.71	0.74	**0.76***	**0.76**	**0.76.**	**0.77.**
	Refined ($\|E\| = 30$)	**0.69.**	**0.74***	**0.76**	**0.77*****	**0.79*****	**0.85*****	**0.86*****
	HELOC ($\|V\| = 23$)							
	CASTLE ($\|E\| = 552$)	**0.75±0.02**	**0.79±0.01**	0.78±0.01	**0.79±0.01**	0.79±0.01	**0.80±0.01**	
	Injected ($\|E\| = 85$)	0.74	0.78***	0.78	0.78***	0.79	0.79***	
MSE ↓	**California** ($\|V\| = 8$)							
	CASTLE ($\|E\| = 72$)	7.05±12.81	2.33±1.39	2.96±4.12	3.86±3.68	4.91±7.41	1.74±1.70	**0.66±0.08**
	Injected ($\|E\| = 31$)	**2.94*****	**2.25**	**1.68****	**1.71*****	**1.51*****	**1.16***	1.02**
	Boston ($\|V\| = 14$)							
	CASTLE ($\|E\| = 182$)	112.0±91.1	21.95±6.84					
	Injected ($\|E\| = 48$)	**86.17*****	**20.45***					

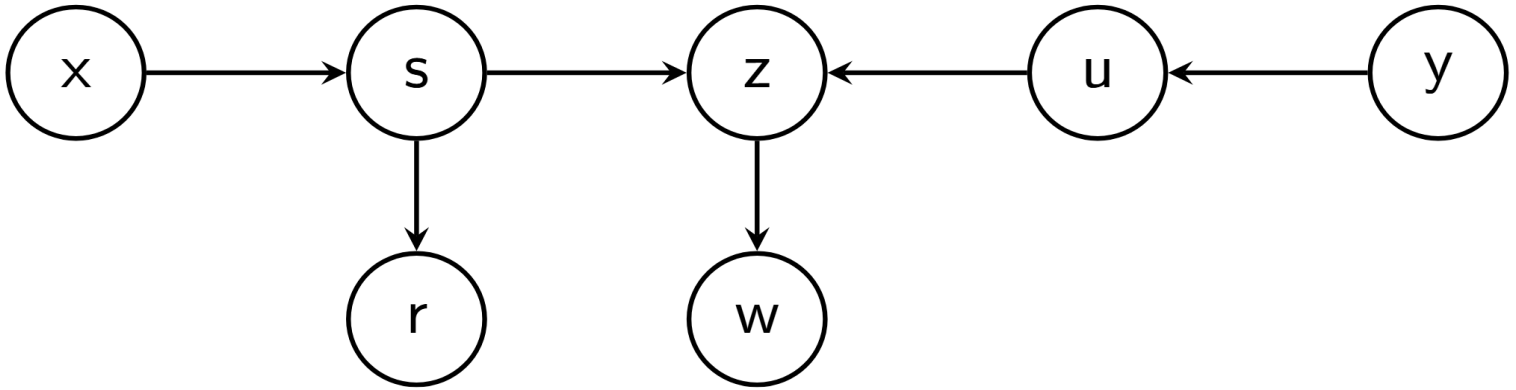
ABAPC Reconstruction Accuracy

on bnlearn datasets (Scutari, 2014)



Connect DAGs and Data: d-Separation

(Pearl 2009)

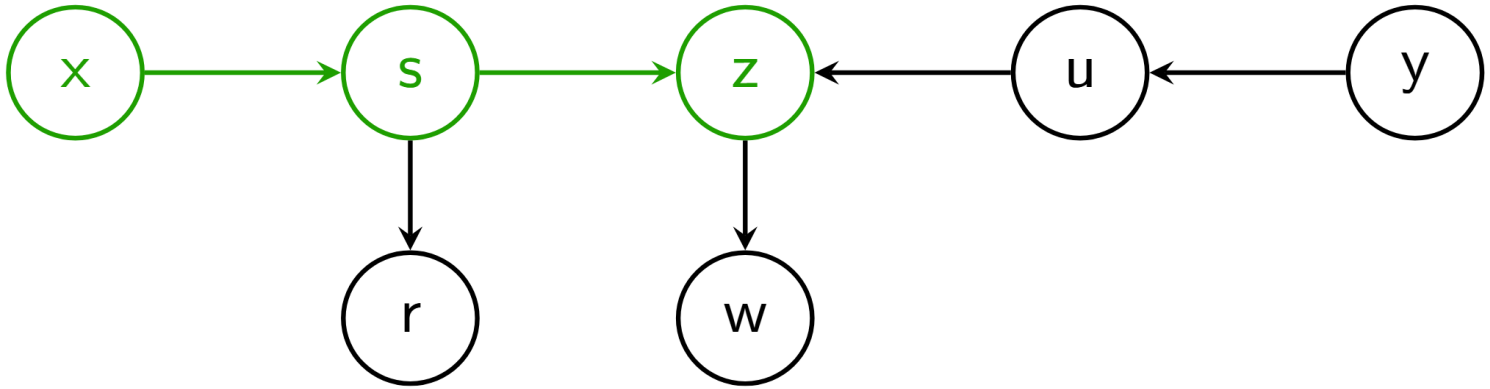


Unconditional

$x \not\perp\!\!\!\perp_{\mathcal{G}} z$

Connect DAGs and Data: d-Separation

(Pearl 2009)

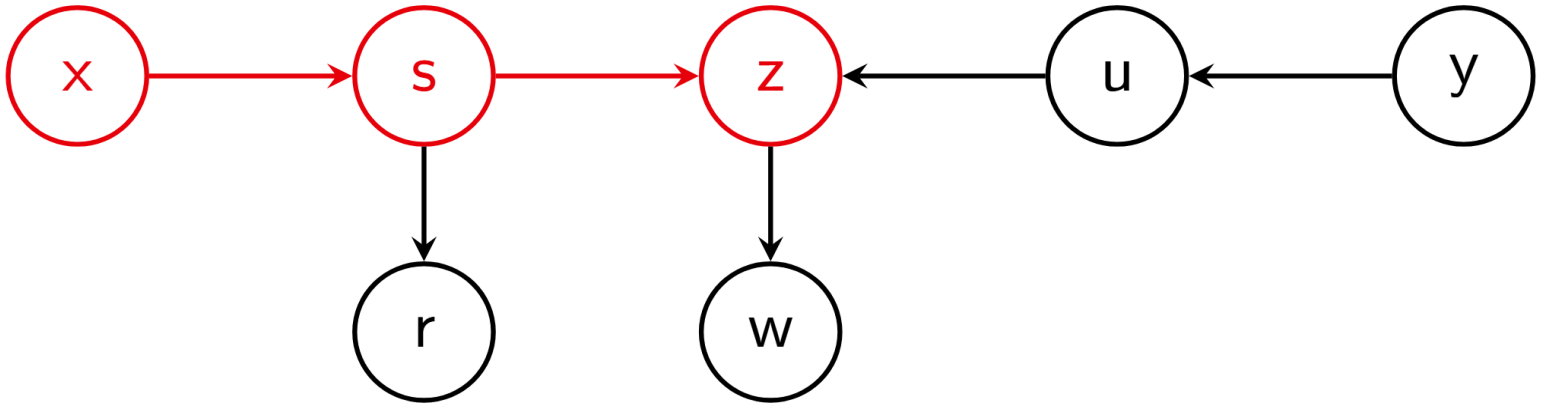


Unconditional

$x \not\perp\!\!\!\perp_{\mathcal{G}} z$ Active

Connect DAGs and Data: d-Separation

(Pearl 2009)



Unconditional

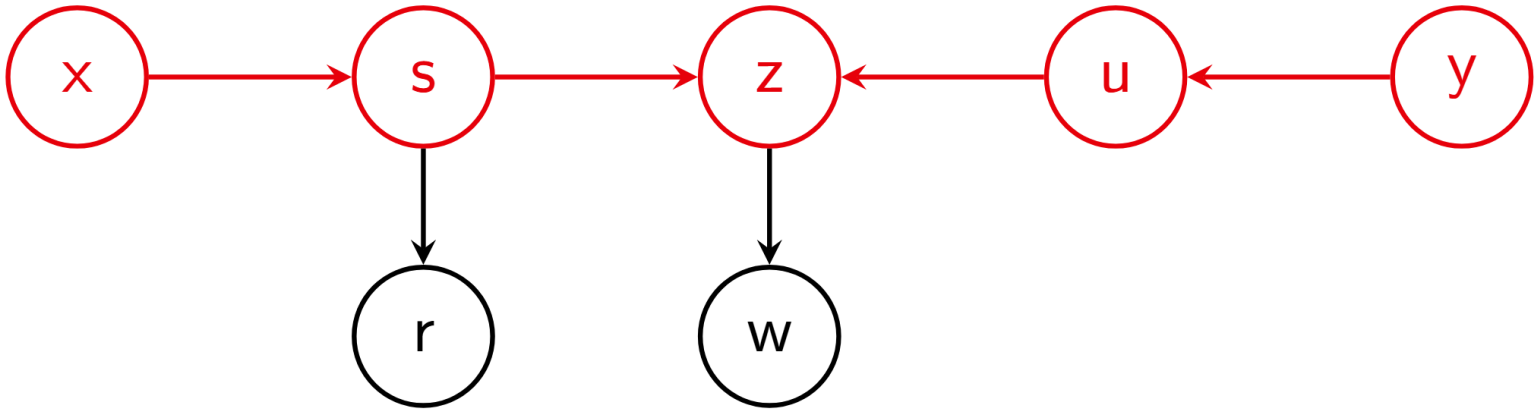
$x \not\perp\!\!\!\perp_{\mathcal{G}} z$

d-separate by conditioning

$x \perp\!\!\!\perp_{\mathcal{G}} z \mid s$ Blocked

Connect DAGs and Data: d-Separation

(Pearl 2009)



Unconditional

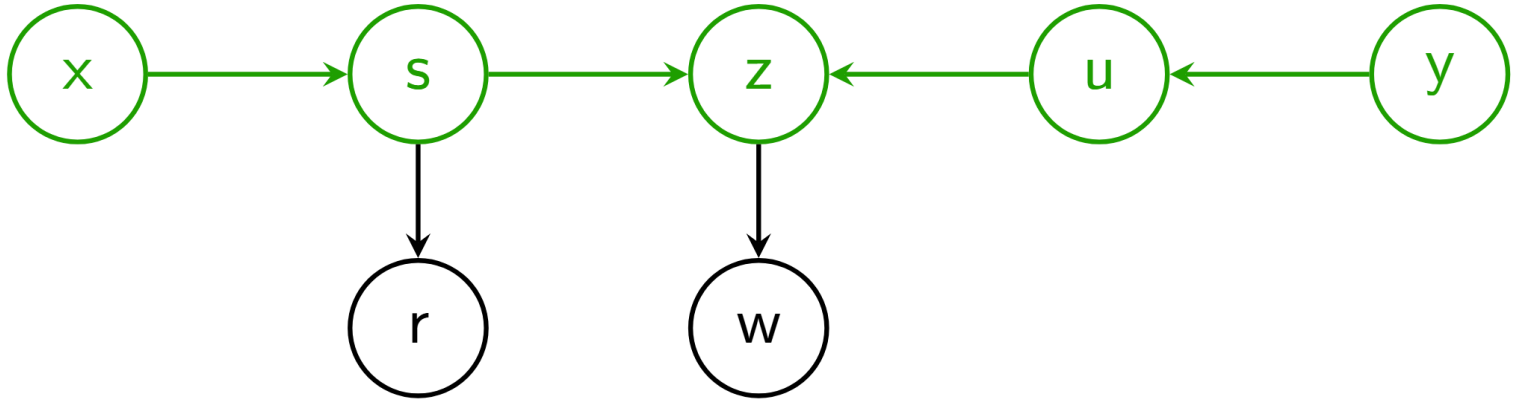
$x \not\perp\!\!\!\perp_{\mathcal{G}} z$

$x \perp\!\!\!\perp_{\mathcal{G}} y$ Blocked

d-separate by conditioning

$x \perp\!\!\!\perp_{\mathcal{G}} z \mid s$

Connect DAGs and Data: d-Separation
(Pearl 2009)



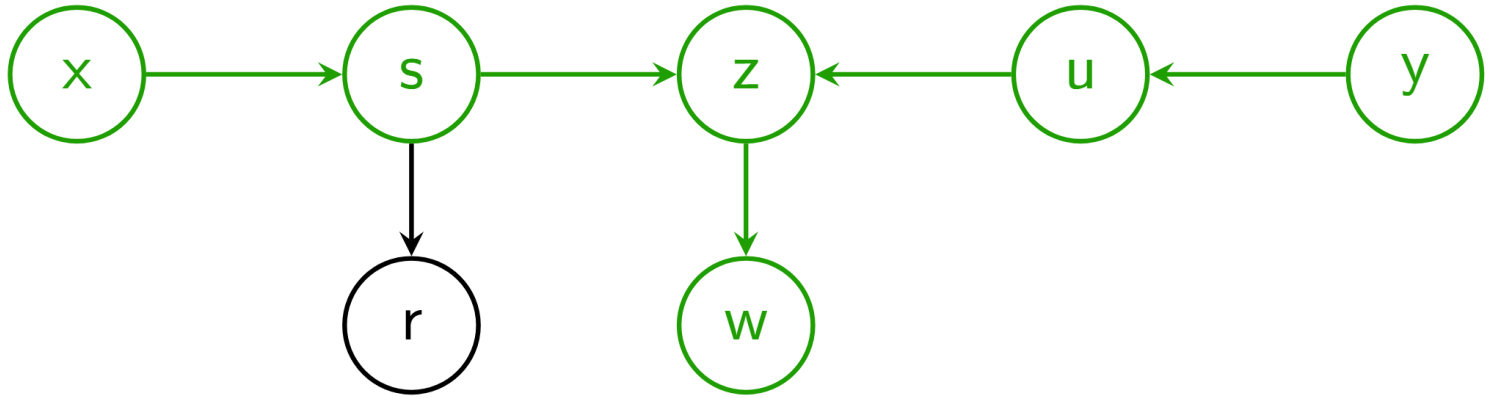
Unconditional
$x \not\perp\!\!\!\perp_{\mathcal{G}} z$
$x \perp\!\!\!\perp_{\mathcal{G}} y$

d-separate by conditioning
$x \perp\!\!\!\perp_{\mathcal{G}} z \mid s$

d-connect by conditioning
$x \not\perp\!\!\!\perp_{\mathcal{G}} y \mid z$ Active

Connect DAGs and Data: d-Separation

(Pearl 2009)



Unconditional

$x \not\perp\!\!\!\perp_{\mathcal{G}} z$

$x \perp\!\!\!\perp_{\mathcal{G}} y$

d-separate by conditioning

$x \perp\!\!\!\perp_{\mathcal{G}} z \mid s$

d-connect by conditioning

$x \not\perp\!\!\!\perp_{\mathcal{G}} y \mid z$

$x \not\perp\!\!\!\perp_{\mathcal{G}} y \mid w$ Active